CHAPTER 3
RADIAL FLOW TO WELLS

Principles & Concepts

As seen in previous chapters, pumping wells can dramatically change groundwater flow velocities and flow directions. Measurement of the magnitude of change is usually quantified by the amount and areal extent of drawdown created by a pumping well. Groundwater moves into a pumping well in response to the lowering of the hydraulic head in the well relative to the surrounding aquifer. This lower hydraulic head in the well is caused by the removal of water from the wellbore by a pump, bucket, or some other device. The removal of water from the well creates a hydraulic gradient in all directions that is inward toward the well (radial flow). In this view, a pumping well is a vertical shaft extending into the aquifer to a depth below the water table (unconfined aquifer) and creates a cone of depression in the water table or potentiometric surface around the pumping well (Figure 3.1).

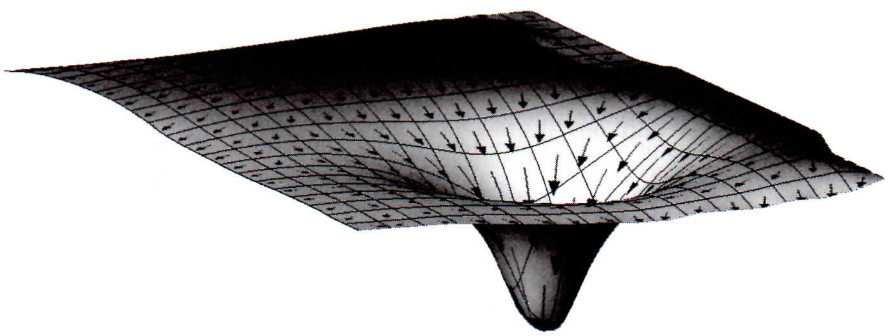


Figure 3.1. Simulated cone of depression showing potentiometric surface around a pumping well and vectors of hydraulic gradient.

A variation of this concept is the special case of a flowing artesian well, whose hydraulic head in the well is naturally higher than the land surface. This relation causes water to flow out of the well without the aid of a pumping device (Figure 3.2). Flowing artesian wells are becoming far more rare as groundwater resources become depleted and potentiometric surfaces decline below land surface elevations.

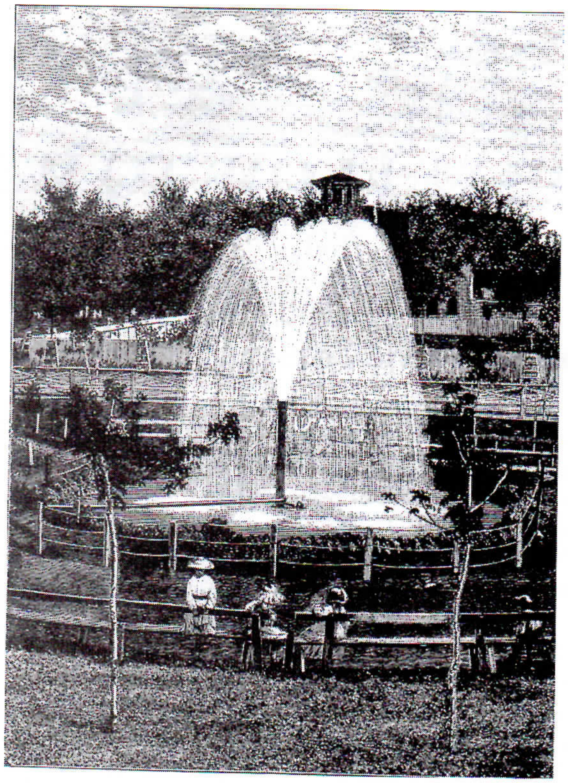


Figure 3.2. Discharge from a flowing artesian well at Prairie Du Chien, Wisconsin, in 1885 (T.C. Chamberlin, 1885).

Groundwater hydrologists have developed several quantitative methods for predicting the drawdown around a pumping well. These equations commonly are used for regulatory compliance to estimate how future pumping will alter groundwater levels. The equations also are commonly used by expert witnesses in court to estimate what effect pumping may have had on water levels, especially in cases where there are few historic water-level measurements. For example, groundwater hydrologists may want to forecast the effect of a new industrial, irrigation, or municipal well on existing groundwater users before the new well is constructed. Another example is the use of these equations to design a dewatering or depressurizing system that will be put to use before excavation of earth for a large foundation under a building, bridge, or tunnel.

This chapter focuses on some of the quantitative methods used to predict the response of an aquifer to an applied pumping stress. In making these predictions, we simulate the response of the aquifer to the pumping stress using a mathematical model of the groundwater flow system. The model is a representation of the groundwater flow system in mathematical terms. It combines the geology of the site with appropriate equations describing the physics of groundwater flow.

The mathematical groundwater flow models presented in this chapter are in the form of radial flow equations having continuous variables in space and time that describe the aquifer, well characteristics, and pumping stress at the site of interest. These mathematical models require simplifying assumptions concerning the geology and the nature of the pumping stress. This simplification is needed because real conditions are more complex than those represented by the equations. For example, to use the mathematical model the geology commonly must be simplified but still be sufficiently realistic to calculate a reasonable aquifer response to the pumping stress for the intended purpose.

Equations describing groundwater flow to a well can be classified according to whether the equation describes steady-state flow or transient flow to the well. Under steady-state conditions, water levels in the aquifer no longer change with time in response to pumping. This lack of change implies that the areal extent of the cone of depression developed around the pumping well is in equilibrium with the pumping stress and that the amount of water produced by the well is balanced by an equal amount of water entering the aquifer as recharge or as leakage across confining layers. If steady-state conditions exist, hydraulic gradients do not vary with time, although they may vary from place to place within the aquifer. Steady-state conditions also imply that there is no net change in the amount of water in storage, because water levels in the aquifer are neither rising nor falling. Thus, steady-state equations of groundwater flow have no storage coefficient term and no time term.

Under transient conditions, water levels change with time. Thus, hydraulic gradients can change temporally and spatially as water moves into or out of storage. Transient equations like the Theis equation contain storage coefficient and time terms and describe the change in drawdown around a pumping well.

The shape of the cone of depression around a pumping well is influenced by both the aquifer itself and by the construction and operation of the well. If the well screen penetrates the entire saturated thickness of aquifer, then the well is said to be fully penetrating. If the well screen penetrates only a portion of the aquifer, then the well is said to be partially penetrating. In fully penetrating wells, equipotential lines are vertical and flow lines are radially horizontal. Thus, the same head is measured at the top as at the bottom of a vertical line through the aquifer. In a partially penetrating well, equipotential lines curve and flow lines must also curve to maintain their orthogonal relation. As seen in Figure 3.3, hydraulic heads near the partially penetrating pumping well can differ substantially depending on the depth of measurement. Therefore, the Theis equation becomes a poor predictor of drawdowns near a partially penetrating pumping well because some of its assumptions are violated.

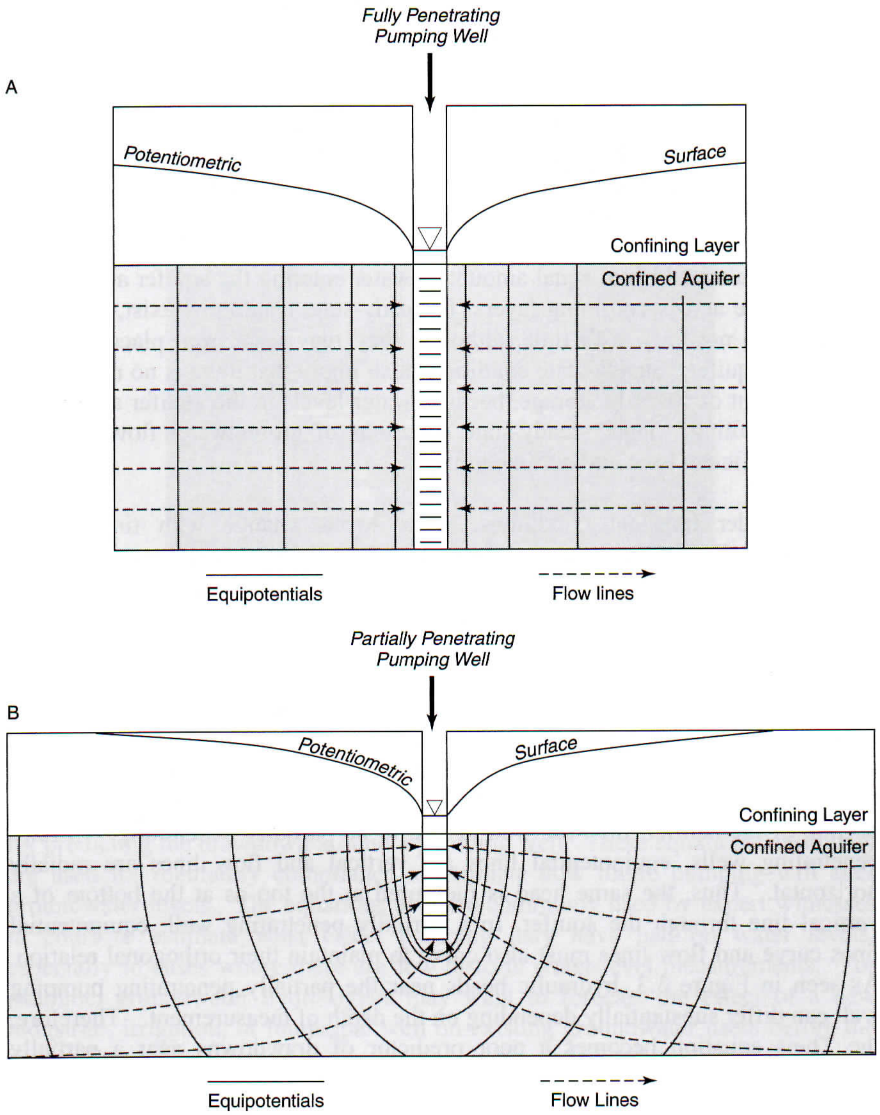


Figure 3.3. (A) A fully penetrating well produces horizontal flow lines and vertical equipotential lines. (B) A partially penetrating well produces vertical equipotential lines and horizontal flow lines only in regions distant from the pumping well. Near the pumping well, the equipotential lines curve in response to the length and position of the partially penetrating well screen. Hydraulic heads will differ with depth where the curvature of the equipotential lines is substantial.

Problem 1

THEIS EQUATION FOR DISTANCE VERSUS DRAWDOWN

Fire Protection Well, River Bend Station Nuclear Power Plant, St. Francisville, Louisiana

Overview

Radial flow of groundwater to a pumping well can be mathematically expressed by combining various forms of Darcy's Law with equations of continuity. Flow to wells in confined or unconfined aquifers and under steady-state or transient conditions can be addressed mathematically. One of the most basic methods and simplest sets of conditions describes a confined aquifer with transient flow to a well that penetrates the entire saturated thickness of the aquifer (i.e., the well is fully penetrating). Theis (1935) was the first to represent these conditions in mathematical terms. Figure 3.4 shows flow to a fully penetrating well in a confined aquifer where the flow is horizontal and radial.

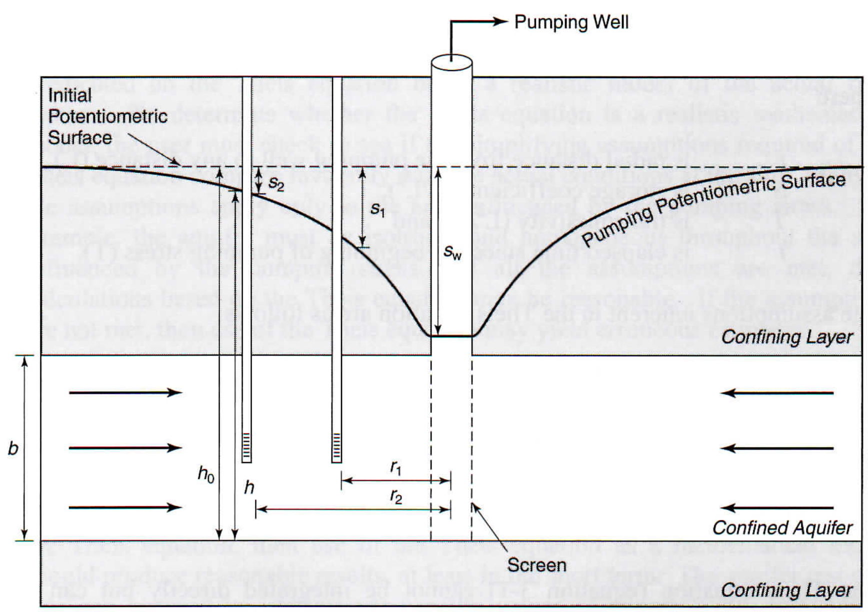


Figure 3.4. Conceptualization of radial flow to a fully penetrating pumping well showing initial and pumping potentiometric surfaces. Two observation wells at radii r_1 and r_2 illustrate the spatial variation in drawdowns s_1 and s_2. The saturated thickness of the aquifer is b and drawdown in the pumping well is s_w.

In 1935 C.V. Theis published his original paper, which described the analogous transient behavior of heat flow to a line sink in an infinite conductive solid and groundwater flow to a well in an infinite aquifer. The mathematical formulation he presented described the amount of drawdown (s) created by a pumping well at any time (t) after pumping commences, at any radial distance (r). This mathematical model became known as the Theis equation and requires certain assumptions about the nature of the aquifer, the pumping rate, and the design of the well. The Theis equation is as follows

$$s = h_o - h = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-u}}{u}\, du \qquad (3\text{-}1)$$

where

s	is drawdown at any time and distance from the pumping well (L),
h_0	is initial hydraulic head at any distance [$t = 0$] (L),
h	is hydraulic head at the same distance after elapsed time [t = t] (L),
Q	is discharge from pumping well (L^3/T),
T	is the transmissivity of the aquifer (L^2/T), and
u	is the Theis equation parameter.

The Theis equation parameter (u) is defined as

$$u = \frac{r^2 S}{4Tt} \qquad (3\text{-}2)$$

where

r	is radial distance from the pumping well to any distance (L),
S	is storage coefficient (L^3/L^3),
T	is transmissivity (L^2/T), and
t	is elapsed time since the beginning of pumping stress (T).

The assumptions inherent in the Theis equation are as follows:

> ***Aquifer*** - *isotropic, homogeneous, uniform thickness, flat lying, infinite in areal extent, overlain and underlain by impermeable layers, releases water instantaneously from storage.*
>
> ***Well*** - *fully penetrates the aquifer, discharges at a constant rate, no borehole storage.*

The Theis equation (equation 3-1) cannot be integrated directly but can be approximated using the following expansion.

$$s = h_0 - h = \frac{Q}{4\pi T}\left[-0.5772 - \ln u + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} + \ \ldots\right] \qquad (3\text{-}3)$$

To simplify the expression, the entire series expansion is usually denoted by the term $W(u)$, which is called the Theis well function. Thus, the Theis equation can be written in a simplified form.

$$s = h_0 - h = \frac{Q}{4\pi T} \; W(u) \tag{3-4}$$

where

$W(u)$	is the Theis well function, and
u	is the Theis equation parameter and is the argument of the function.

We use the series expansion (equation 3-3) of the integral to solve the Theis equation with *Excel*. (See Appendix D for a list of $W(u)$ versus u values). This first exercise entails programming the Theis Well Function $W(u)$ into the "*Distance-Drawdown*" worksheet and solving the Theis equation for drawdown (s) at various values of time (t) and distance (r). Because each successive term in the infinite series becomes smaller and smaller, $W(u)$ is approximated by accounting for a finite number of terms. The separate terms are then summed to obtain an approximate value of the integral. The number of terms we use differs between problems but is usually six to ten. We limit the approximation to ten terms in this exercise to illustrate the nature of the Theis well function.

The Theis equation is a powerful tool for estimating the effects of future and previous pumping stress on nearby water levels. These predictions are predicated on the Theis equation being a realistic model of the actual flow system. To determine whether the Theis equation is a realistic mathematical model, the user must check to see if the simplifying assumptions required of the Theis equation compare favorably with the actual conditions at the site. Many of the assumptions apply only to the area influenced by the pumping stress. For example, the aquifer must be isotropic and homogeneous throughout the area influenced by the pumping stress. If all the assumptions are met, then calculations based on the Theis equation may be reasonable. If the assumptions are not met, then use of the Theis equation may yield erroneous estimates.

Another check on the suitability of using the Theis equation to make predictions can be made with the results from a multiple-well aquifer test. This test is a controlled field experiment performed to determine whether observed time-drawdown behavior matches theoretical time-drawdown behavior based on the Theis equation. If the observed response matches the response predicted by the Theis equation, then use of the Theis equation as a mathematical model should produce reasonable results, at least in the short term. The aquifer test data also can be used to estimate site-specific values of transmissivity and storage coefficient in other mathematical models that are used to make predictive calculations. More information about aquifer tests and their interpretation is found in Chapter 5.

An additional aquifer characteristic used to describe the ability of water to move through an aquifer is transmissivity. Transmissivity (T) is the rate of flow under a unit hydraulic gradient through a cross section of the aquifer that has a unit width for the entire saturated thickness of the aquifer. It is determined by the following equation.

$$T = K\,b \tag{3-5}$$

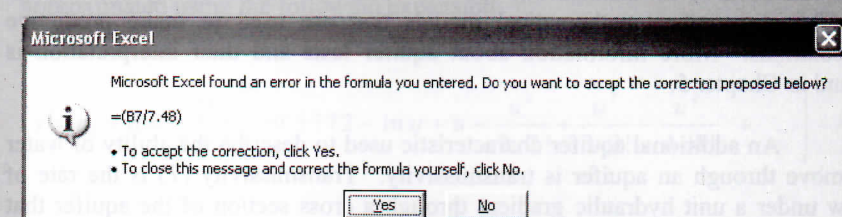
where

T	is the transmissivity of an aquifer (L^2/T),
K	is the hydraulic conductivity (L/T), and
b	is the saturated thickness of the aquifer (L).

It is tempting to use the Theis equation to compute the drawdown in the pumped well itself using a radius representing the size of the wellbore or the size of the sand filter or gravel pack. This approach may greatly underestimate the drawdown that actually occurs in the pumped well because of additional drawdown caused by frictional losses created as water moves through the sand filter or gravel pack and across the screen. The Theis equation does not account for this additional drawdown, which is collectively known as well loss. It is possible to estimate the amount of additional drawdown in the pumped well owing to well losses for a given pumping rate on the basis of analysis of a step-drawdown test performed in the pumping well (Kruseman and de Ridder, 1990).

Excel Tips

- FACT(n) is an intrinsic *Excel* function that takes the factorial of the number "n." Use FACT(n) to increase the efficiency of programming the W(u) function.
- PI() inserts the constant π into an equation.
- LN(n) calculates the natural logarithm of the number "*n*."
- SUM() sums the cells designated within the parentheses.
- Cell anchor: $column$row anchors the cell (column, row) in a formula. Performing a relative copy will maintain the anchored cell within the formula and change only the other non-anchored cells in the formula (e.g., =$D$12+D2 will anchor cell D12 and not cell D2 during a relative copy).
- Exponent symbol: ^ raises a number to the designated power (e.g., 3^2 = 9).
- Other intrinsic *Excel* functions are listed and explained by clicking the f_x button in the Toolbar.
- Using the F2 key on a cell with a formula will highlight the reference cells used in the formula. This allows you to more easily evaluate the accuracy of the formula programmed into the cell.
- *Excel* will give an error message if a programming error occurs such as illustrated below with missing parentheses. However, *Excel* will not necessarily give an error message if the formula is entered incorrectly.



- "#DIV/0!" will appear in cells if a division by zero error has occurred. #DIV/0! displays if the denominator in any formula is zero including reference cells that may also contain an error. Thus, one "#DIV/0!" error may propagate through the worksheet to multiple cells. Check your formulas for accuracy to avoid these errors.
- Save your work often to avoid losing information.

Parameter	***Definition***
b	Saturated thickness of the aquifer (L)
h_0	Initial hydraulic head at any distance (L)
h	Hydraulic head at the same distance after elapsed time (L)
K	Hydraulic conductivity of the aquifer (L/T)
Q	Discharge from pumping well (L^3/T)
r	Radial distance from the pumping well to any distance (L)
S	Storage coefficient (L^3/L^3)
s	Drawdown at any time and distance from the pumping well (L)
T	Transmissivity of the aquifer (L^2/T)
t	Elapsed time since the beginning of the pumping stress (T)
u	Theis equation parameter
$W(u)$	Theis well function

Table 3.1 Parameter Definition Table – Chapter 3, Problem 1

Problem 2

THEIS EQUATION FOR TIME VERSUS DRAWDOWN

Dewatering System, River Bend Station Nuclear Power Plant, St. Francisville, Louisiana

Overview

The temporary lowering of the water table in shallow aquifers to facilitate excavation of rock or sediment before construction of foundations for buildings, bridges, and construction of tunnels is a common practice. This problem examines the dewatering of an unconfined shallow aquifer in Louisiana for the construction of a nuclear power plant. The River Bend Station power plant was constructed northwest of Baton Rouge, Louisiana, outside the town of St. Francisville. In order to excavate the 41-acre footprint of the foundation for the buildings to a depth of approximately 100 feet, the water table needed to be lowered 65 feet to facilitate excavation and construction in dry conditions. The water table was lowered by installing 44 high-capacity pumping wells around the perimeter of the excavation and discharging the pumped water to a local bayou (see the Overview of Problem 2 on the CD).

The Theis equation can be used to predict the long-term behavior of water levels in both confined and unconfined aquifers if certain assumptions are valid. These assumptions include the Theis assumptions regarding the aquifer and the pumping well that are described in Problem 1 of this chapter and for the long-term behavior of water levels in unconfined aquifers. In unconfined aquifers, the Theis equation can be a reasonable predictor of long-term water levels. The late-time drawdown response to a pumping well in an infinite unconfined aquifer is consistent with the Theis assumptions except that the late-time response is controlled by the specific yield, not by the coefficient of storage (Kruseman and de Ridder, 1990).

Well Interference

Pumping a well causes water levels in the aquifer to draw down and form a conical depression in the water table of an unconfined aquifer or in the potentiometric surface of a confined aquifer.

If two pumping wells are in proximity, their individual cones of depression may overlap. In this case, the calculated drawdowns from each well are additive in areas where the cones overlap, such that the total drawdown at any location in the aquifer is the sum of that produced by the two pumping wells (Figure 3.6). The additive nature of drawdowns is known as well interference and refers to the composite drawdown produced by two or more pumping wells. For example, if well A created 3 feet of drawdown halfway between wells A and B, and well B created 2 feet of drawdown at this same point, then the composite (total) drawdown would be 5 feet at this halfway point.

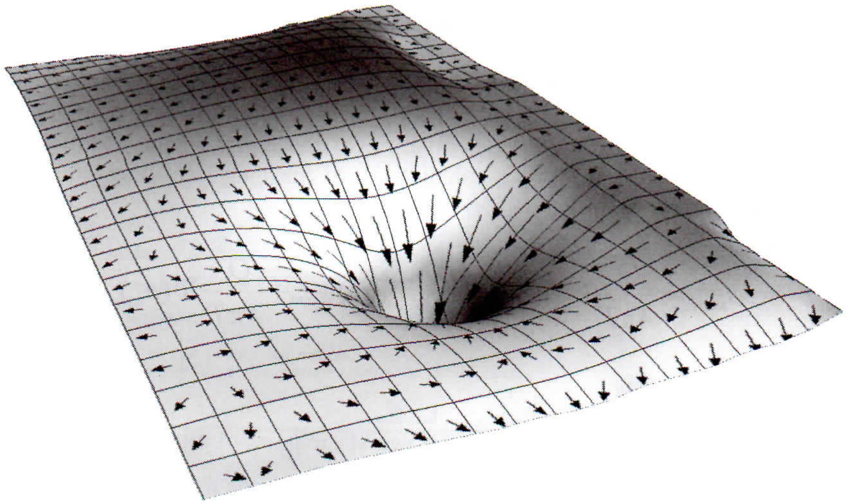


Figure 3.5. Water-table configuration showing a cone of depression forming around a pumping well. The magnitude of the hydraulic gradient is represented by the arrows increasing in length closer to the pumping well.

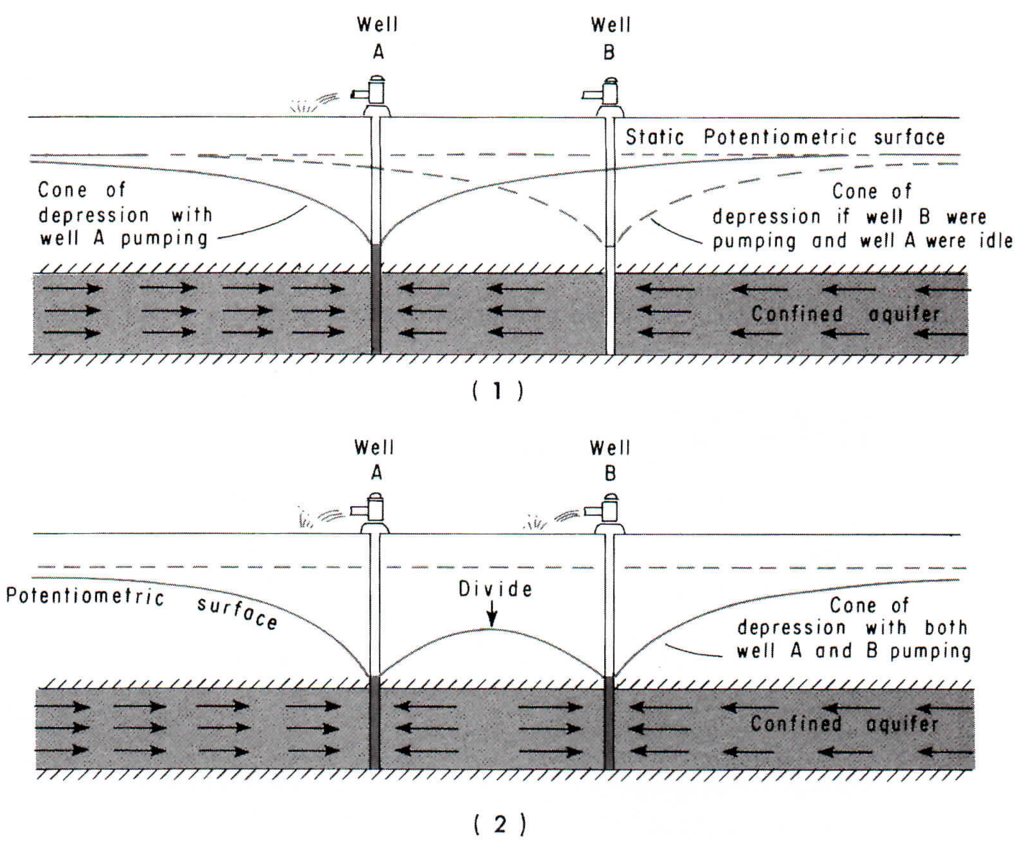


Figure 3.6. Well interference between two pumping wells (A and B) showing the additive nature of drawdown: (1) shows individual cones of depression around each well and (2) shows the composite cone of depression produced by both wells (modified from Heath, 1998).

The Theis equation can be used to compute the size and shape of a composite cone of depression from two or more pumping wells. These calculations become more complicated if several wells are pumping, the wells are at different distances from the points of interest, and they are pumping at different rates. The following general form of the Theis equation is used to make these composite drawdown calculations of the long-term behavior in an unconfined aquifer.

$$s_{total} = \frac{Q_1}{4\pi T}W(u_1) + \frac{Q_2}{4\pi T}W(u_2) + ... + \frac{Q_n}{4\pi T}W(u_n) \qquad (3\text{-}6)$$

where

s_{total}	is the composite drawdown at the point of interest produced by n pumping wells (L),
n	is the number of wells,
Q_n	is the discharge rate of each pumping well (L^3/T),
T	is the transmissivity (L^2/T), and
u_n	is the Theis equation parameter for each well.

$$u_n = \frac{r_n^2 S_y}{4Tt} \qquad (3\text{-}7)$$

where

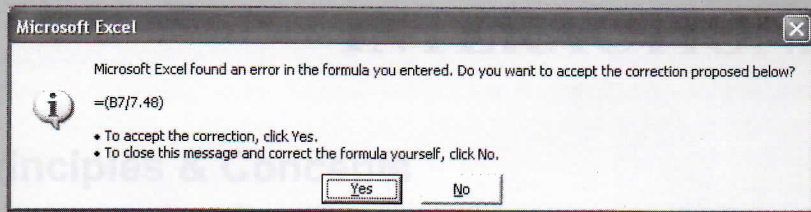
r_n	is radial distance from each pumping well to the point of interest (L),
S_y	is specific yield (L^3/L^3), and
t	is elapsed time since the beginning of the pumping stress (T).

As you can no doubt appreciate, spreadsheets can greatly assist in making these calculations.

Excel Tips

- FACT(n) is an intrinsic *Excel* function that takes the factorial of the number "n." Use FACT(n) to increase the efficiency of programming the W(u) function.
- PI() inserts the constant π into an equation.
- LN(n) calculates the natural logarithm of the number "*n*."
- SUM() sums the cells designated within the parentheses.
- Cell anchor: $column$row anchors the cell (column, row) in a formula. Performing a relative copy will maintain the anchored cell within the formula and change only the other non-anchored cells in the formula (e.g., =$D$12+D2 will anchor cell D12 and not cell D2 during a relative copy).
- Exponential symbol: ^ raises a number to the designated power (e.g. 3^2 = 9).
- Other intrinsic *Excel* functions are listed and explained by clicking the f_x button in the Toolbar.

- Using the F2 key on a cell with a formula will highlight the reference cells used in the formula. This highlighting allows you to more easily evaluate the accuracy of the formula programmed into the cell.
- *Excel* will give an error message if a programming error occurs such as illustrated below with missing parentheses. However, *Excel* will not necessarily give an error if the formula is entered incorrectly.



- "#DIV/0!" will appear in cells if a division by zero error has occurred. "#DIV/0!" displays if the denominator in any formula is zero, including formulas in reference cells that may also contain an error. Thus, one "#DIV/0!" error may propagate through the worksheet to multiple cells. Check your formulas for accuracy to avoid these errors.
- "#NUM" will appear in the $W(u_n)$ terms prior to programming the input parameters in the worksheet. This designation should resolve once the worksheet programming is complete.
- Save your work often to avoid losing information.

Parameter	*Definition*
b	Saturated thickness of the aquifer (L)
h_o	Initial hydraulic head at any distance (L)
h	Hydraulic head at the same distance after elapsed time (L)
n	Number of wells
Q_n	Discharge from each pumping well (L^3/T)
r_n	Radial distance from the pumping well to any point (L)
S	Drawdown at any time and distance from the pumping well (L)
s_{total}	Composite drawdown at the point of interest produced by n pumping wells (L)
S_y	Specific yield (L^3/L^3)
T	Transmissivity (L^2/T)
t	Elapsed time since the beginning of the pumping stress (T)
u_n	Theis equation parameter for each well
$W(u)$	Theis well function

Table 3.2 Parameter Definition Table – Chapter 3, Problem 2

Problem #1: Theis Equation for Distance versus Drawdown

Fire Protection Well, River Bend Station Nuclear Power Plant, St. Francisville, Louisiana

The extraordinary safety concerns at a nuclear power plant require that cooling water for the plant be from a different source than water for fire protection. At River Bend station, cooling water is obtained from the Mississippi River at an intake structure about a mile from the plant, where the water is clarified and then pumped to the plant. Groundwater from a well completed to a depth of 1,800 feet into the Tertiary Zone 3 aquifer is used for fire protection.

River Bend station is one of eight nuclear power plants operated by Entergy Operations, Inc. in the southern and northeastern United States. River Bend station is located 24 miles north of Baton Rouge, Louisiana in West Feliciana Parish. The 3,300-acre site abuts the Mississippi River on the west. The power plant itself is located about a mile to the east on a bluff overlooking the river (Figure 3.1.1). River Bend station is capable of producing 980 megawatts of electricity. In 2003, it produced 7,600,000 megawatt hours that were used by more than 2 million customers in Louisiana, Mississippi, Texas, and Arkansas.

Figure 3.1.1. Location of River Bend station power plant outside of St. Francisville, Louisiana, on the upland adjacent to the Mississippi River.

As part of the licensing documentation required by the U.S. Nuclear Regulatory Commission, the amount of drawdown produced during a fire at River Bend station in the nearest well also completed in the Tertiary Zone 3 aquifer has to be estimated. The Theis equation can be used to make this estimation.

Based on performance and analysis of an aquifer test in the fire protection well, a maximum sustainable yield of 900 gallons per minute can be used in this calculation and site-specific values of transmissivity and storativity are 35,200 gallons per day per foot and 0.00072, respectively.

The nearest well also completed in the Tertiary Zone 3 aquifer was operated by the Crown-Zellerbach Corporation and located 7,500 feet away, on the other (west) side of the Mississippi River (Figure 3.1.2). Like many similar engineering calculations, unknown values of parameters, such as the duration of pumping the fire protection well, are addressed conservatively by using a value that overestimates (in this case) the amount of drawdown in the Crown-Zellerbach well. Hence, the fire protection well at River Bend station was assumed to operate for the entire 40-year design life of the power plant.

Questions for Problem #1: Answer the questions 1-4 in Excel file ch3_p1.xls.

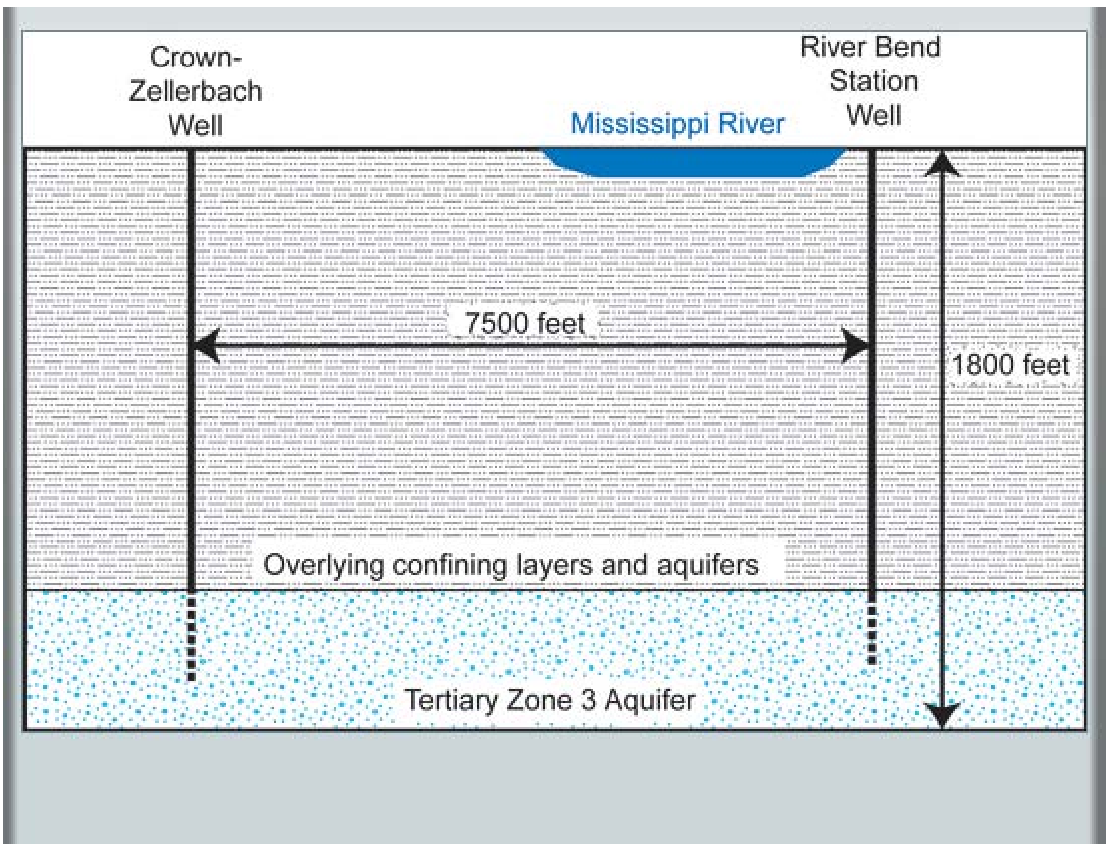


Figure 3.1.2. Schematic diagram of Crown-Zellerbach well and fire protection well at River Bend station.

Problem #2: Theis Equation for Distance versus Drawdown

Dewatering System at River Bend Station Nuclear Power Plant, St. Francisville, Louisiana

As seen in Problem 1 of this chapter, the River Bend station nuclear power plant was constructed near the Mississippi River in Louisiana. During the six-year construction project, an excavation covering 41 acres and up to 100 feet deep (Figure 3.2.1) was created to construct the foundation for the reactor building, the cooling water towers, radwaste building, and other structures. To complete this task, the shallow unconfined aquifer in the surficial loess, Port Hickey silts and clays, and the Citronelle Formation sands and gravels had to be dewatered and the water table lowered 65 feet. A temporary dewatering system was installed to maintain the water table below the bottom of the excavation so that construction could occur under dry conditions. The dewatering system contained 44 wells (see Figure 3.2.2) spaced around the perimeter of the excavation. Each well is approximately 145 feet deep, equipped with a turbine pump powered by a diesel engine and capable of pumping 700 gallons per minute through 12-inch diameter screen (see Figure 3.2.3).

Figure 3.2.1 The 41-acre 100-foot deep excavation at River Bend station created for construction of the nuclear power plant and other structures had bench cuts at the edge of excavation. The access ramp is seen on the bottom left of the photograph.

Figure 3.2.2 Close up of bench cuts in silt material along the edge of the excavation. Three dewatering wells are seen along the perimeter of the excavation.

Figure 3.2.3 One of the 44 dewatering wells surrounding the excavation at River Bend station.

The combined discharge from all 44 wells was released into a nearby bayou that flowed to the Mississippi River. The dewatering system operated during two separate construction phases over a six-year period. The maximum combined discharge from the 44 wells was more than 22,000 gallons per minute (31.7 million gallons per day) but averaged about 7,700 gallons per minute (11 million gallons per day) (see Figure 3.2.4).

The cones of depression from the 44 dewatering wells interfere with each other to create a composite cone of depression. Because drawdown is linearly additive, the size and shape of the composite cone of depression can be estimated using the Theis equation. The spreadsheet problem uses the Theis equation to predict the long-term behavior of the water table under various pumping scenarios (see Reference Book). Using the Theis equation is a reasonable approach to this problem that entails making estimates of flow system behavior several years into the future because the period of delayed yield in most unconfined aquifers only persists from several hours to a few days. After this short period of time, the long-term time-drawdown behavior can be characterized with the Theis equation using a value of specific yield instead of the coefficient of storage.

Questions for Problem #2: Answer the questions 1-5 in Excel file ch3_p2.xls.

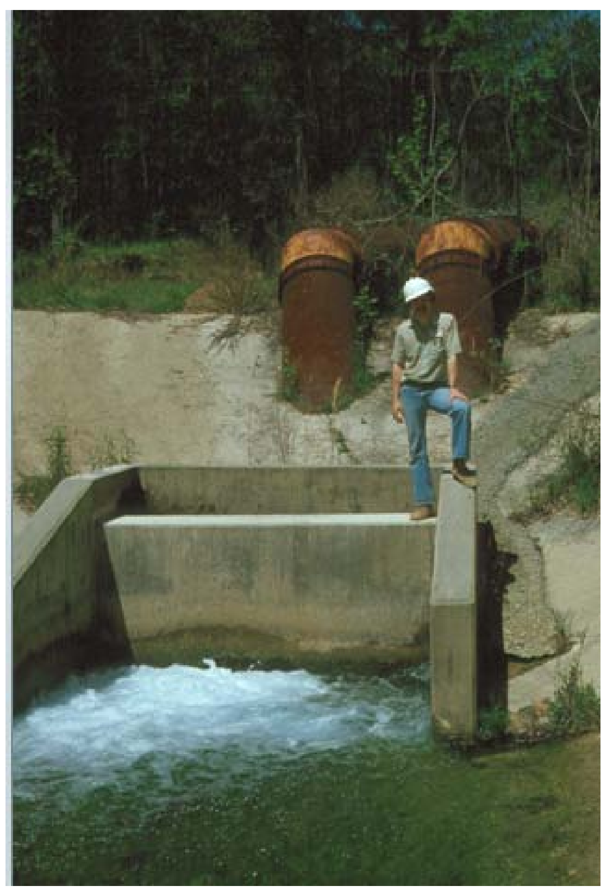


Figure 3.2.4 Combined discharge of the 44 dewatering wells into a local bayou.